A Cellular Automata Models of Evolution of Transportation Networks

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A CELLULAR AUTOMATA MODELS OF EVOLUTION OF TRANSPORTATION NETWORKS

We present a new approach to modelling of transportation networks. Supply of resources and their influence on the evolution of the consuming environment is a principal problem considered. We present two concepts, which are based on cellular automata paradigm. In the first model SCAMAN (Simple Cellular Automata Model of Anastomosing Network), the system is represented by a 2D mesh of elementary cells. The rules of interaction between them are introduced for modelling of the water flow and other phenomena connected with anastomosing river. Due to limitations of SCAMAN model, we introduce a supplementary model. The MANGraCA (Model of Anastomosing Network with Graph of Cellular Automata) model beside the classical mesh of automata, introduces an additional structure: the graph of cellular automata, which represents the network pattern. Finally we discuss the prospective applications of the models. The concepts of future implementation are also presented.
Keywords: *Cellular Automata, transportation network, anastomosing network*

MODELE EWOLUCJI SIECI TRANSPORTOWEJ WYKORZYSTUJĄCE AUTOMAT KOMÓRKOWY
Prezentujemy nową koncepcję modelowania sieci transportowych. Głównym, rozważanym przez nas problemem jest dostarczanie zasobów do konsumującego je środowiska oraz ich wpływ na jego ewolucję. Proponujemy dwie koncepcje oparte na paradygmacie automatów komórkowych (Cellular Automata). W pierwszym z proponowanych modeli, nazwanym SCAMAN (Simple Cellular Automata Model of Anastomosing Network), system jest reprezentowany przez klasyczną dwuwymiarową siatkę automatów. Zdefiniowane zostały reguły interakcji między automatami w celu modelowania przepływu wody oraz innych zjawisk związanych z rzekami anastomozującymi. Ograniczenia modelu SCAMAN spowodowały, że zdefiniowaliśmy drugi model. W modelu MANGraCA (Model of Anastomosing Network with Graph of Cellular Automata) oprócz klasycznej siatki automatów wprowadzona została dodatkowa struktura: graf automatów komórkowych, który reprezentuje wzór sieci rzecznej. Przedyskutowane zostały możliwości przyszłych zastosowań proponowanych modeli oraz koncepcje ich implementacji.
Słowa kluczowe: *automaty komórkowe, sieci transportowe, sieci anastomozujące*

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1. Introduction

The network structures in nature, in most cases, are connected with transportation or communication functions. They can be created to perform realistic transportation tasks such as circulation of blood in vessels, data flow in computer networks or traffic flow in road networks. Their creation can be also a result of various natural or artificial processes. In all these cases the network fulfills the same function: provides the tracks along which the objects are moved from one place to the other.

The transportation networks play very important role both in nature and in the area of human activity. No wonder that so much research is done on its various aspects. A considerable amount of work is focused on modelling and analyzing the motion in networks. The results find many applications e.g. in design and monitoring of computer networks and architecture, in traffic management [1, 2] urban planning [3] and many more.

The network can be also considered as a complex dynamic system, evolving according to specific rules, with the number of factors, which influence on the final pattern. The studies on such a system may follow to build a numerical model of a particular type of the network, which allows to predict its future behaviour or to observe its past evolution.

River is an example of the natural transportation networks. It transport solid materials eroded in upper course to lower areas. Also solved organic resources are collected and redistributed along the whole course. Thus, we can treat a river system as a transportation network which supply surrounding environment. Changes in environment influence on river network topology. This interaction makes river networks a very interesting dynamic complex system with various properties. Some of them are general for broader range of networks created by e.g. polymers [4], vascular system [5], Internet or wiring of the brain [6].

The studies on creation and evolution of river networks have focused on branching and meandering rivers. Broad overview of models of branching networks can be found in [7]. Generally, we can distinguish three main class of models: statistical, deterministic and lattice models. The two first class represents continuum approach (nonlinear differential equations are applied), while lattice models exploit mainly Cellular Automata approach.

For anastomosing river there have not been proposed yet any numerical models, which could be a starting point for serious studies on their dynamics. The main reason of such the situation is the rare appearance of anthropogenically undisturbed anastomosing river systems and the unclear mechanisms which govern its functionality.

The purpose of this paper is to present a conception of a new approach to modelling of evolution of transportation networks. Main aspect, which we are trying to simulate is an interaction between transportation network and consuming environment. We have started from the model of anastomosing rivers but the other examples of such the interactions can be pointed out (see sec. 3). Our another goal is to provide a mathematical model, which could be useful for studies on properties of such networks structure. We hope also that our method can find application in computer graphics to modelling of realistic landscape.

Section 2 describes the phenomena of anastomosing river and all factors and mechanisms which influence its creation and evolution. In section 3 we touch upon the interaction of transportation network and environment in more details. Following sections presents the conception of the models, we introduce. In the last section conclusions are discussed.

2. Anastomosing rivers

An anastomosing network term describes a blundering, irregular network composed of branches and junction (divergent and convergent) knots. The network may have hierarchical and fractal structure. Anastomosing river can be an example of such a network in nature. Complete overview of that type of rivers and factors as well as phenomena that built such patterns can be found in [8] and [9].

The anastomosing river is the fourth main type of river beside the straight, meandering and braided. In classical river system we can distinguish three main segments: upper erosional convergent zone, middle transit zone and lower divergent outlet ([7], chapter 1). Anastomosing rivers are developed in middle zone on a flat, wide areas with a very small slope (about 5 cm per 1 km). The main reason of the creation and the existence of anastomosing systems is plant vegetation. Plants receive necessary resources — nutrients (ions of nitrogen, phosphorus and potassium) from water. Products of their decay are accumulated in the interchannel areas as a peatbogs. The accretion (vertical accumulation) of peat layer is accompanied by sedimentation of solid material on the bottom of channels.

Plants activity is also the main factor contributing to the formation of new channels. The rate of plant grow is controlled by nutrient supply. The nutrient's flux gradually disappears perpendicularly to the channel due to suction of plants root system (see Fig. 1a). Gradual progress of nutrient deficit, provides starvation of peat-forming plants and decrease of accretion of peats. The accretion of bottom sediments and peats along the channel banks follows to rising up of water level (see Fig. 1b).

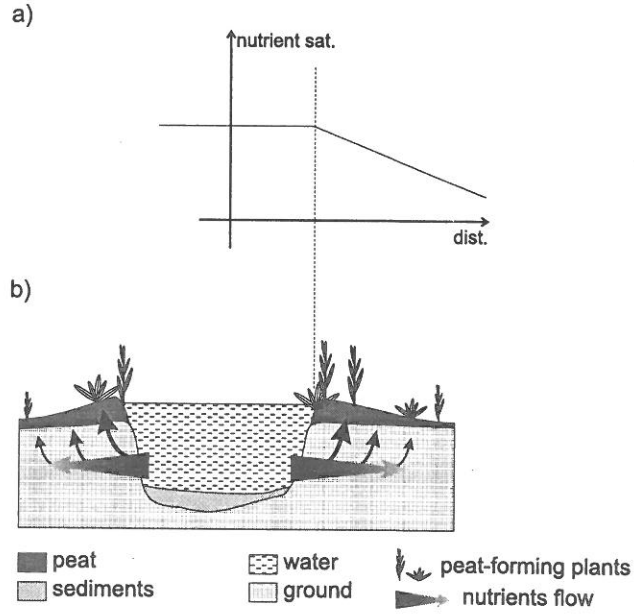

Fig. 1. Plot of distribution of nutrients concentration in ground-water *vs* distance from channel bank (a), the cross-section of channel (b)

Such a situation is thermodynamically unstable. The water plants overgrowing the riverbeds cause jams in current channels and initiate feedback mechanisms which subsequently lead to the avulsion (i.e. the relatively sudden displacement of a part or the whole of river channel [10]). A new branch is initiated above the jam zone and may join the main channel somewhere downstream. The route is determined by the topography and flows through the lowest area. The evolution of newly created channels proceeds similarly to the evolution of the main stream. Finally, a very complex system of hierarchically branching, joining and interconnecting channels is created (see Fig. 2).

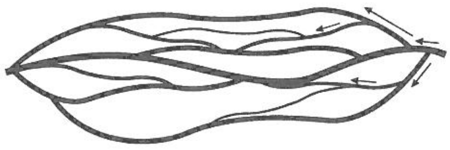

Fig. 2. Anastomosing network

3. Transportation networks

Anastomosing river system can be seen as a transportation network in a consuming environment. River supply nutrients which are disseminating to the ground. The nutrients are one of factors controlling the vegetation of peat-forming plants. Other factors, such as sunlight, can be considered as spatially constant. Therefore we assume that the nutrients level control the rate of accretion of peat layer.

The thickness of the peat layer indicate how the environments are supplied. The rate of the peat grow is fastest on the area nearby channels and decrease with the grow of distance from the water. However, the system drives towards the state in which the whole environment is equally supplied. When the new channel is created, gravity forces to flow through a lower area. The peat layer on this area starts growing. The old channels, slowly overgrowing, stop supplying the resources and the development of peatbog becomes restrained near this channels.

In nature as well as in the area of human activity other phenomena analogous to the creation and evolution of anastomosing rivers can be pointed out, e.g. angiogenesis of blood vessels [11, 12], development of road systems or computer networks. Such the networks are also built-up to supply certain resources to a consuming environment.

The other types of networks (e.g. branching) can also fulfills such the functions. Branching networks does not transport material through the environment, but only deliver some resources. We can also reverse this principle and consider situation in which transportation networks transport resources the outside of the environment. Such situation we observe on root system, which takes various resources from soil and transport them to the cells.

4. Cellular Automata approach

Since Mandelbrot has shown the fractal nature of many natural phenomena [13], it became clear that in many situations it is not necessary to use complex and sophisticated mathematical paradigms. Fractal geometry supplemented by chaos theory shows that complex natu-

ral structures and phenomena often can be a result of simple interacting processes. This observation points out that the utilization of mathematical instruments, which allow to express this processes in the simplest way, can be successful.

Cellular Automata (CA), introduced by von Neumann and Ulam in late 40's and significantly developed by Wolfram [14] in 80's, fulfil the conditions of simplicity very well. CA can be treated as a mathematical idealization of physical systems with discrete time and space. The rules of local interactions govern the evolution of the system. The CA are useful in simulations of dynamic systems for which global evolution depends on local interactions between elementary parts. Although CA have not fulfilled all expectations, list of their applications still increases. They have been applied to modelling macroscopic transport phenomena e.g. water flow [15], lava flow [16, 17], snow deposition [18], heat conduction [19] as well as urban traffic [20, 21]. CA were also applied to river networks modelling [22, 23, 24], however limited to branching river networks only.

5. SCAMAN model

The SCAMAN (Simple Cellular Automata Model of Anastomosing Network) model exploits the CA paradigm in a classical way. The landscape is represented by a regular 2D mesh of automata. At each cell the following parameters are specified: the altitude, the amount of water, and the thickness of a peat layer. Water is supplied to the systems from "source" cells, in which the amount of water is filled up in each iteration. Other special cells are defined as "outflow". For these cells, the amount of water is decreased in each iteration. The amount of water in the remaining ("ordinary") cells can change depending on transition rules between them. In general, water is disseminated between cells and their neighbours, according to the obvious rule, which says that water flows from the cell with the higher altitude to the cell with the lower altitude.

The cells containing water are the sources of nutrients, which are distributed to their neighbours. The algorithm of nutrient distribution must provide an expected gradient of nutrients saturation (see Fig. 1a) around the cells with water.

In each step, the thickness of peat layer is increased. The rate of peat growth depends on nutrients saturation. Another phenomena, which must be taken into consideration is evaporation. This factor cannot be omitted, because it may follows to situations, in which the isolated areas of water, may exist forever. In order to prevent such effect, some amount of water must be constantly removed from the system.

5.1. Formal definition of automaton in SCAMAN model

CA is defined as

$$CA_{SCAMAN} = \langle Z^2, A_I, A_O, X, Q, \sigma \rangle,$$

where:

- Z^2 – the set of cells with integer coordinators in 2-dimensional Euclidean space;
- $A_I \subset Z^2$ – the set of "sources";
- $A_O \subset Z^2$ – the set of "outflow";
- X – defines the neighbourhood of a given cell by the finite set of 2-dimensional vectors;

Q – the finite set of states of elementary automaton:

$$Q = Q_w \times Q_g \times Q_t \times Q_n,$$

where the substates are as follows:

Q_w – the amount of water in the cell,

Q_g – the altitude of the cell,

Q_t – the thickness of peat layer,

Q_n – the amount of nutrients in the cell;

$\sigma: Q^n \rightarrow Q^{n+1}$ – the deterministic state transition function for the cells in Z^2; the σ function is composed of three steps performed for each cell:

1. compute the water distribution to the neighbourhood,
2. compute the nutrients distribution to the neighbourhood,
3. update the thickness of the peat layer;

the detailed specification of each step is given in the next section.

5.2. The outline of proposed algorithm

As was mentioned in the previous section, the transition function σ is composed of three steps, which refers to considered phenomena.

Main loop of the algorithm is as follows:

```
for each timestep do:
    distribute_water()
    distribute_nutrients()
    update_peat_thickness()
    remove_water()
```

Water redistribution is calculated by using the algorithm presented in [15]. The flow from central cell to each neighbouring cell is calculated according to the altitude and amount of water in the cell. The thickness of peat layer is stored separately, therefore the algorithm should take as a whole altitude, the sum of elementary altitude and peat thickness.

Nutrients distribution is also calculated using Cellular Automata algorithm.

1. All the cells, which contain the water have a maximum saturation of nutrients.
2. For all the remaining cells, the nutrient saturation must be calculated:
   a) the maximum saturation is searched in neighborhood of cell i, $N(i)$;
   b) if founded value *vmax* is greater than zero, the saturation in cell i become equal to *vmax* – *GN*.

The *GN* stands for nutrients gradients (the value depends on mesh size), $N(i)$ is a set of neighbors of cell i. This procedure provides an expected distribution of nutrients.

The increase of the peat layer depends linearly on nutrients level in a particular cell. In each iteration for each cell in the mesh the thickness of peat layer is increasing according to nutrients level in this cell.

Very small amount of water is removed from each cell (which contain water) in each timestep of simulation in order to model the evaporation.

6. MANGraCA model

For realistic simulations using SCAMAN model, high performance computers are necessary. First, the landscape must be represented in high resolution to gain more complicated pattern of channel networks. Second, as our tests show, with the model of water flow [15], the propagation of changes of states of automata through the mesh is very slow. It's mean that the simulation using SCAMAN may take a lot of time steps and must be performed on very large meshes (we estimate that mesh cannot be smaller than 1000 × 1000 cells). For this reason we are introducing another model.

The idea of this model is based on a remark that the network of anastomosing river is similar to a graph structure. As it is seen in Figure 3, the river network can be represented by a graph in which the vertices stand for the points of branching and joining of channels, nodes and the edges represent the channels.

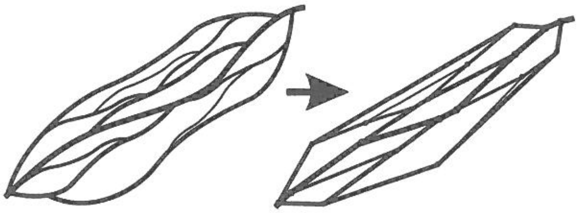

Fig. 3. Anastomosing network to graph

In order to take an advantage of CA, this concept was modified. The graph is constructed on a classic mesh of automata, by selecting some cells and establishing additional relations of neighbourhood (see Fig. 4b). In such a graph, the cells are represented by vertices and relations by the edges. Additionally, since the flow direction is taken under consideration, the graph has to be directed. Such the graph represents the network of channels.

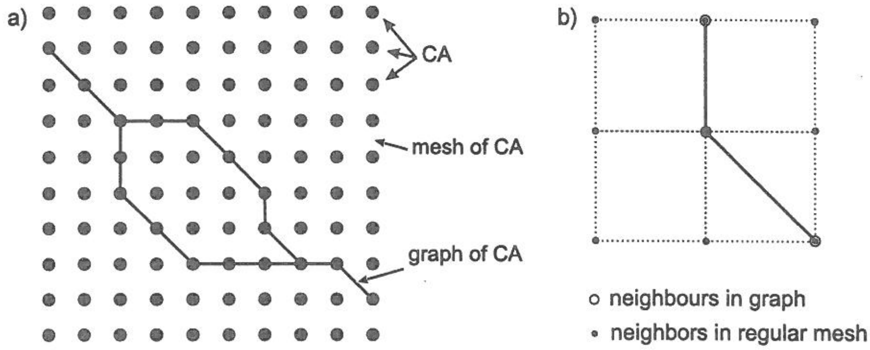

Fig. 4. Graph of CA: a), b) see text for details

Some of the parameters, which describe the state of an automaton such as the altitude, the thickness of peat layer, the level of nutrients, are identical as in SCAMAN model. There are two additional parameters specified, which describe basic properties of a network: throughput and flow rate. Initially in each cell these parameters are inactive until the cell is added to the graph. Then they are set to initial values, which change in successive time steps of simulation according to the algorithm. The throughput of initial path is given as a starting parameter of simulation. It is modified by random to simulate the overgrowing of the channels. Flow rate in the first cell is initialized to an arbitrary value (less than throughput!). Then it is propagated to the other cells currently pertaining to the graph. In each time step the flow is newly recalculated.

Decrease of the throughput occurs very slowly. Occasionally, in randomly chosen cell, the throughput is decreased suddenly below the level of flow in this cell. This corresponds to creation of jam in the river bed. Thus, the new channel must be created. It starts in a cell standing behind the cell in which the jam occurs. The throughput in the new channel is initialized to about 25% of the throughput of the superior channel. The route of new channel is determined by the shape of the landscape. The procedure of routing is a crucial to our algorithm. Generally, the route may be calculated using the steepest descent method. Such simple algorithm may fail encountering a local minimum, therefore we should define additional functionality, that prevents procedure from unexpected blocking. Although, the water cannot moves up, we can assume that the flowing water may cover the small roughness and hills. More detailed specification will be given later.

When the throughput parameter becomes close zero, flow in this cell stops. It is mean that water cannot flows this channel. Branch is removed from the network.

Cells belonging to the graph exerts an influence also on their neighbors in a classic 2D mesh, being a source of nutrients. They are distributed to the other cells using the same algorithm as in the SCAMAN. The thickness of peat layer is changing according to the amount of nutrients. The growth of the peat layer causes the changes in shape of the landscape, influencing on the route of channels, which will be created in future.

6.1. Formal definition of automaton in MANGraCA model

Cellular Automaton is defined as

$$\mathrm{CA}_{\mathrm{MANGraCA}} = < Z^2, a_i, X_K, G_{\mathrm{CA}}, Q, \sigma >,$$

where:

- Z^2 – the set of cells with integer co-ordinators in 2-dimensional Euclidean space;
- $a_i \in Z^2$ – the cell, which supply water to the system;
- X_K – defines the neighbourhood of a given cell on the classical 2D mesh by the finite set of 2-dimesional vectors;
- G_{CA} – the directed graph of cellular automata constructed over the mesh; $G_{\mathrm{CA}} = (V, E)$, where $V \subset Z^2$ is a set of vertices and E is a set of edges;
- Q – the finite of states of elementary automaton:

$$Q = Q_g \times Q_p \times Q_n \times Q_t \times Q_f,$$

where the substates are as follows:

Q_g – the altitude of the cell,
Q_p – the thickness of peat layer,
Q_n – the level of nutrients in the cell,
Q_t – the throughput of the cell,
Q_f – the value of flow intensity in the cell;

$\sigma: Q^n \rightarrow Q^{n+1}$ – a deterministic state transition function for the cells in Z^2; the σ function is composed of three steps performed by each cell:
1. computing the nutrients distribution;
2. updating the thickness of peat layer;
3. updating the throughput parameter;

full specification of each step is given in the next section.

7. The outline of proposed algorithm

The algorithm is very simple:

```
trace(source)
for each timestep do:
    update_flows();
    distribute_nutrients();
    update_peat_thickness();
    modify_throughput();
    if throughput(i) < flow(i) then
    trace(i)
```

Function trace realizes the procedure of tracing the route of new channel. Procedure resemble the simulated annealing schema, however there are some important differences.

1. For two cells, lately joined to graph G_{CA}, the direction of movement is calculated (see Fig. 5).
2. The line of movement determine the set of potential successors S.
3. From S, the cell with lowest altitude is selected – x
4. If the altitude of cell x is less than altitude of the cell x^k, x^p is joined to graph.
5. Otherwise:
   a) the cell x^p is randomly chosen from S,
   b) x^p is joined to graph.

Algorithm stops if the cell x^p already belongs to graph G_{CA}. It correspond to joining the new channel to the other channel of river network.

The flow rate is arbitrarily set in source cell. Function (update_flows( )) propagate the flow parameter to the remaining cells in graph. Propagation is governed by two obvious rules:

1) the sum of flow rates, which enter to the cell must be equal to sum of flow rates, which leaves this cell;
2) the division of flow rate in divergent cell depends on throughput in its successors.

The distribution of nutrients (distribute_nutrients( )) is calculated in the same way as in SCAMAN model, but the sources of nutrients are the cells belonging to graph. Also the thickness of peat layer is updated (update_nutrients( )) as in SCAMAN.

In the cells, which belongs to graph G_{CA}, in each timestep of simulation, the throughput parameter is decreased. It correspond to process of sedimentation and overgrowing the river bed. The rate of decrease is very small. Occasionally in randomly chosen cells, the throughput is decreased much faster. It corresponds to creation of jams in the river beds.

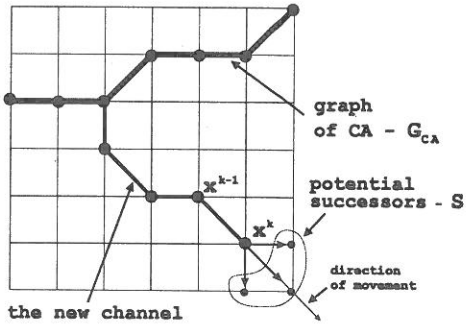

Fig. 5. Procedure Trace ()

The first step of simulation consists in calculating the route of initial channel. The procedure trace() starts from arbitrarily chosen cell. Next the following operations are processed in the loop:

1. The flow rate is updated.
2. The nutrient distribution is recalculated.
3. The value of peat thickness is updated.
4. The throughput is updated in the cells belonging to graph.
5. If there a cell in which the throughput is less then a flow rate:
   a) the new path is initiated in the predeceasing cell,
   b) function trace() calculates the route of the new path.

The creation of complex graph is expected as shown in Figure 6c.

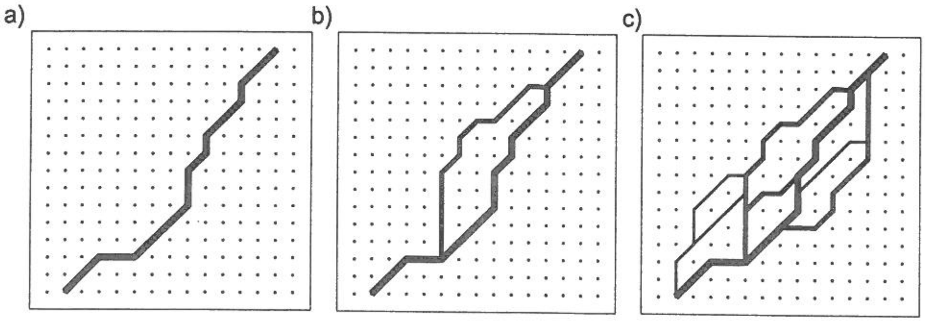

Fig. 6. The algorithm with graph of CA a)–c) see text for details

8. Conclusions and future works

The conclusions and the area of future works can be enumerated as follows:

- Our models takes into consideration only the principal phenomena which contribute to the creation and evolution of anastomosing rivers. The precise modelling including all the phenomena and influencing factors is impossible without considerable complication of the models.
- We assume the existence of two self-dependent objects: the network structure and the environment in which this network fulfils the transportation function, supplying or draining off given resources. The model of mutual influence of the transportation network and the consuming environment is general for larger group of phenomena (we have pointed out a few of them in section 3), while the mechanism of creation of new channels, does not. The application of our approach to modelling other phenomena must be preceded by studying the mechanisms, which govern their evolution.
- Anastomosing rivers have not been included to studies on dynamics of river networks until now. Therefore the verification of results obtained is only qualitative. Anastomosing river has a different topology that the more common branching river networks, so we should estimate if the methods, which are applied for those type of river, are adequate for anastomosing networks. The studies on the real river patterns are necessary. They can allow for quantitative comparison of the real pattern and the results produced by numerical model.
- The idea of only local interactions, which is the basis of SCAMAN model give us a very precise tool for modelling local water distribution. The terrain must be represented by using high resolution meshes. Moreover, the size of mesh must be very large in order to model large anastomosing systems. On such the large system, all the changes propagate very slowly. Therefore the realistic simulations require high-performance computer systems.
- The approach utilized in MANGraCA let us reduce the size of mesh and accelerate the simulation, at the cost of precision. By using MANGraCA we cannot yield information about local water distribution, changes in landscape and shapes of river bed. In return we can obtain complicated pattern of channels in incomparable shorter time, than SCAMAN model.

  The lack of information about local water distribution makes independent application of MANGraCA model in visualization of anastomosing river rather limited. In other application: the studies on evolution of transportation networks, studies on properties of anastomosing network and visualization of network structure in which the spatial dimensions of branch are not important, MANGraCA is much more useful than SCAMAN.
- The MANGraCA produces the global network pattern, while the SCAMAN gives more precise information about local distribution of water in terrain. A hybrid model which employees some elements of both concepts should allow for simulating multiresolution structure. The implementation of two-level model in which the global evolution of the system is performed by using MANGraCA while its local behaviour is modelled by SCAMAN would be a reasonable step in this direction. This concept assumes that

the large grain computation are performed employing the MANGraCA model. The higher resolution can be obtained by performing SCAMAN model. The substantial problem of this approach lies in „gluing" of the two models, which work on different spatio—temporal scales. The problems of multiscale models and their implementations are currently the area of intensive studies (see e.g. [25]).

- One of the most important features of CA is the ability of relatively easy implementation of the efficient parallel algorithms within SPMD (Single Program, Multiple Data) model [26]. SCAMAN, which exploits cellular automata paradigm in classical way, may be very easy implemented in parallel. In Parallel-SCAMAN, regular mesh of automata can be shared between nodes of parallel machine. The nodes process independently assigned parts of the mesh. After each step of simulation they must exchange the information about the state of boundary cells.

  Parallel-MANGraCA is more difficult to implement. Significant amount of communication involved by graph based algorithms, makes them very inefficient, especially on architecture with distributed memory e.g. clusters. Additionally, the irregular structure of graph of CA, generates the problems with data decomposition between computational nodes.

  A mixed parallel-sequential algorithm, would be an issue. The operations connected with graph of cellular automata (e.g. update_throughput, update_flows – see sec. 7) will be preformed sequentially on node. The operations connected with regular mesh of automata (e.g. update_nutrient_distribution, update_peat_thickness) are processed in parallel on node (in the same way as in SCAMAN model). This method require more communication than Parallel-SCAMAN. In each timestep of simulation communicate, not only one with another, but also with the. This can make the Parallel-MANGraCA less efficient than its sequential equivalent, especially when the small or medium size of mesh will be applied.

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